Mathematics: analysis and approaches
Standard Level
Paper 1

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The line L passes through the points $P(-5, 4)$ and $Q(11, -8)$. Find the equation of the line perpendicular to L that passes through the midpoint of $[PQ]$. Write your answer in the form $y = mx + c$.

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2. [Maximum mark: 6]

Let $f(x)=\frac{1}{2x+1}$ and $g(x)=2x-3$. Given that $h(x)=(f\circ g)(x)$, find:

(a) $h(x)$; [2]

(b) $h^{-1}(x)$. [4]

3. [Maximum mark: 5]

The first derivative of a function g is given by $(x-4)^3$.

(a) Find the second derivative of g. [2]

(b) Write down the value of $g''(4)$. [1]

(c) The x-coordinate of point A on the graph of g is 4. Explain why A is **not** a point of inflexion. [2]

4. [Maximum mark: 7]

Given that $\log_3 2 = x$ and $\log_3 5 = y$, express each of the following in terms of x and y.

(a) $\log_3 20$ [2]

(b) $\log_3 \left(7\frac{13}{16}\right)$ [2]

(c) $\log_5 8$ [3]

5. [Maximum mark: 6]

Consider the infinite series $1+\ln x+(\ln x)^2+\cdots$.

(a) Find the values of x such that the series converges. [3]

(b) Find the value of x such that the series converges to 2. [3]

6. [Maximum mark: 6]

Solve the equation $8\sin x\cos x = \sqrt{12}$, for $0 \le x \le \frac{\pi}{2}$.

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

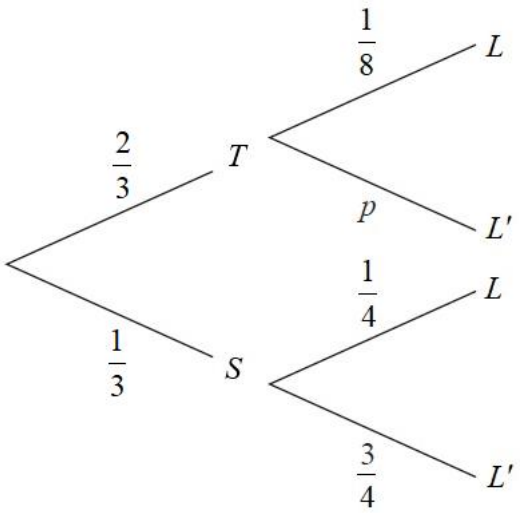
7. [Maximum mark: 14]

Sophie travels to school by taking a taxi (T) or by riding a scooter (S). On any given school day, the probability she travels by taxi is $\frac{2}{3}$ and the probability she travels by scooter is $\frac{1}{3}$.

If Sophie travels by taxi, her probability of being late to school is $\frac{1}{8}$.

If Sophie travels by scooter, her probability of being late to school is $\frac{1}{4}$.

This information is represented by the following tree diagram.



(a) Find the value of p. [2]

(b) Find the probability that Sophie will travel by taxi and be late for school. [2]

(c) Find the probability that Sophie will be late for school. [3]

(d) Given that Sophie is late for school, find the probability that she travelled by taxi. [3]

Sophie will go to school on Monday, Wednesday and Friday next week.

(e) Find the probability that Sophie will be late exactly twice during next week. [4]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

Consider the function $f(x) = 2x^2 + 4x - 5$.

(a) Write f in the form $f(x) = a(x-h)^2 + k$. [4]

(b) For the graph of f in the form $f(x) = a(x-h)^2 + k$.

(i) Write down the coordinate of the vertex.

(ii) Write down the equation of the axis of symmetry.

(iii) Write down the coordinates of the y-intercept.

(iv) Find the coordinates of both x-intercepts. [8]

(c) Hence, sketch the graph of f. [2]

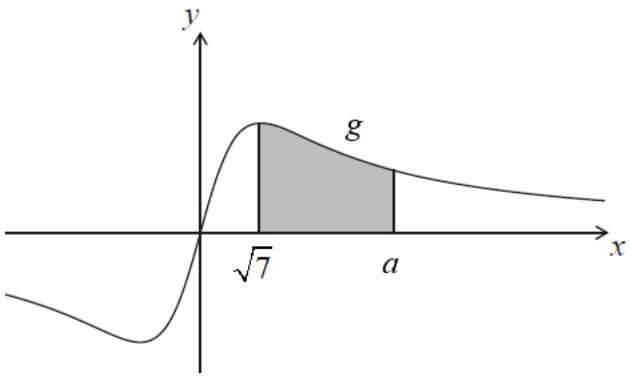
9. [Maximum mark: 16]

The function g is defined by $g(x) = \frac{3x}{x^2+7}$.

(a) Show that $g'(x) = \frac{21-3x^2}{(x^2+7)^2}$. [5]

(b) Find $\int \frac{3x}{x^2+7}\,dx$. [4]

The diagram below shows a portion of the graph of g.



(c) The shaded region is enclosed by the graph of g, the x-axis, and the lines $x = \sqrt{7}$ and $x = a$. This region has an area of $\ln 8$. Find the value of a. [7]